A QUANTITATIVE METHOD FOR THE PREDICTION OF RAINFALL PATTERNS

by

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ABSTRACT

Under certain assumptions, the vertical motion in the low and middle troposphere is directed upward in regions where the relative vorticity decreases downstream along the streamlines at 300 mb, and inverse. Computations covering sixteen observed charts show that in general at least some rainfall reaches the ground where upward motion should be present following the above. Fourteen charts giving actual 24-hour predictions show that this approach can be used to aid in the forecasting of precipitation patterns.

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Shortly after the end of World War I, J. Bjerknes and Solberg (1) published their famous paper on the formation of rain. Since then, forecasters in many parts of the world have availed themselves of the precipitation models given in their daily practical routine. The fact that the models have survived for thirty years in forecast offices is outstanding testimony of their value.

Nevertheless, as pointed out from time to time (cf. 2), departures from the models on occasion may be substantial. It is appropriate, therefore, to inquire into the circumstances under which these departures occur, and thus to facilitate their prediction. We can follow one possible avenue by relating precipitation areas to the wind patterns of the high

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troposphere that have received much attention in recent years. Starrett (3) has found good correlations between the jet stream and precipitation. More recently, Norquest (4) noted that over the United States nearly all precipitation areas of large size -- covering several states or more -- are connected with the jet stream and that even the majority of smaller patchy areas -- up to the size of a state -- are linked to the high-level flow. Thus the proposed line of inquiry is not an illogical one. Moreover, methods have now been developed (5, 6) that permit calculation of the prognostic wind field aloft for twenty-four hours, and these may be capable of extension in time. If we can find some fairly unique relation between precipitation and the upper flow, we would then be in a position to directly compute the areas where precipitation should not be predicted from the high-level prognostic chart.

Following previous articles on the relations between the vorticity field of the atmosphere and large scale vertical motion (7, 8, 9), we shall write an expression that relates the vertical motion of the lower troposphere to the vorticity field of the upper troposphere. We do not desire an elegant, complete mathematical expression but a tool that can be used readily by forecasters who have the equipment of conventional forecast offices at their disposal.

Derivation: In the areas of general cyclonic precipitation, mass convergence in the lower troposphere gives way to divergence at some upper level. This we know not from direct observation of vertical motions but from the fact, known since the early days of meteorology, that in the vast majority of cases the surface pressure either falls or changes slowly in precipitation zones. Although the height h_o at which the reversal takes place, may vary widely (10), it generally lies below the levels used for the upper wind prognosis. We shall try to ascertain the sign of the vertical motion at this elevation.

In its complete form, the equation of continuity reads

$$\frac{\partial \rho}{\partial t} + V\cdot\nabla\rho + \rho\nabla\cdot V + \frac{\partial}{\partial z}(\rho w) = 0, \qquad (1)$$

where ρ is the density, V the horizontal velocity, w the vertical velocity, z the vertical coordinate, and ∇ denotes the horizontal gradient. Integrating from the height h of the surface of nondivergence to the top of the atmosphere or, for practical purposes, to some level well inside the stratosphere, perhaps 100 mb,

$$\rho w\Big]_h = \int_h^\infty \left(\frac{\partial \rho}{\partial t} + V\cdot\nabla\rho\right) dz + \int_h^\infty \rho\nabla\cdot V\, dz. \qquad (2)$$

We shall now neglect the first term on the right hand side and assume that the mass divergence above h results from the velocity divergence. Such an assumption, of course, can find its justification only in the final result. We have reason, however, to suspect that the assumption is not a bad one. The local derivative of density with time is known to be smaller by orders of magnitude than the other terms. More important, the integration here performed usually includes parts of troposphere and stratosphere. It is well known that the sign of local density variation and of density advection reverses with warm advection in the stratosphere, and the inverse also holds. Thus the net density advection between a tropospheric surface of non-divergence and the higher stratosphere may be close to zero.

With the above assumption:

$$\rho w\Big]_h = \int_h^\infty \rho \nabla\cdot V\, dz. \tag{3}$$

Given a perfect upper wind prognosis, no further steps are required. In general, however, this is impractical. The definition of divergence is

$$\nabla\cdot V = \frac{\partial V}{\partial s} + V\frac{\partial\alpha}{\partial n},$$

where s is the horizontal coordinate along the streamlines, n the normal axis pointing to the left of the streamlines, and α the angle between the s axis and some fixed coordinate, usually taken along the latitude circles. In most cases, the two terms comprising the divergence are of opposite sign, i.e. the geostrophic wind is a first approximation to the field of motion. $\nabla\cdot V$ is a small residual between two large terms, and even the best wind prognosis is not apt to give this residual with anything approaching accuracy.

It is desirable, therefore, to eliminate the divergence from equation (3). We can do this by introducing the vorticity equation, a technique employed in the literature quoted earlier. The definition of the relative vorticity ζ_r is

$$\zeta_r = -\frac{\partial V}{\partial n} + V\frac{\partial\alpha}{\partial s},$$

where $\frac{\partial\alpha}{\partial s} = K$ is the curvature of the streamline radius. By observation, the two terms of this expression very often have the same sign. Thus upper wind prognoses can give a fair distribution of the vorticity field, even though it is a derivative quantity of the wind field. We shall proceed on this basis.

The vorticity equation reads

$$\frac{1}{\zeta}\frac{d\zeta}{dt} = -\nabla\cdot V, \tag{4}$$

where ζ is the vertical component of the absolute vorticity and d/dt is the substantial derivative. Inserting (4) in (3),

$$\rho w\Big]_h = -\int_h^\infty \frac{\rho}{\zeta}\frac{d\zeta}{dt}\, dz. \tag{5}$$

For evaluation, we must break the substantial derivative into its partial components. By definition,

$$\frac{d\zeta}{dt} = \frac{\partial \zeta_r}{\partial t} + V\frac{\partial \zeta}{\partial s} + w\frac{\partial \zeta_r}{\partial z}$$

In this expression, the term $w\frac{\partial \zeta_r}{\partial z}$ is likely to be very small, since in the integral over the whole upper portion of the atmosphere a contribution can come about only by fluxes through the bottom. Similarly $\frac{\partial \zeta_r}{\partial t}$ usually is small compared to $V\frac{\partial \zeta}{\partial s}$. This statement expresses the observation that in general the upper air patterns move much more slowly than the wind itself. For the most part then it is reasonable to retain the advective term alone. Occasionally, $\frac{\partial \zeta_r}{\partial t}$ can be large, namely when the center of an intense jet stream is displaced from one side of a forecast district to the other. Then, the relative vorticity of the jet stream level can change by as much as 2f to 3f in 24 hours, where f is the Coriolis parameter. We have not investigated cases of this kind. It may become necessary to do so in future work.

With the foregoing considerations,

$$\rho w_h = -\int_h^\infty \frac{\rho}{\zeta} V \frac{\partial \zeta}{\partial s}\, dz. \tag{6}$$

The absolute vorticity $\zeta = f + \zeta_r$ and its gradient $\partial\zeta/\partial s = \partial f/\partial s + \partial \zeta_r/\partial s$. Throughout the middle latitudes $\partial f/\partial s \sim 10^{-5}$ sec^{-1} (5° lat)$^{-1}$. Near the level of strongest wind $\partial \zeta_r/\partial s$ has the magnitude of 10^{-4} sec^{-1} (5° lat)$^{-1}$ in the vicinity of jet stream axes. Although this value will be reduced by the vertical integration, the gradients of relative vorticity far overshadow those of the Coriolis parameter. In regions where they do not, it is best not to apply the technique here outlined, because of the assumptions made in this paper and the limitations of the wind prognosis. In our estimate, the gradient of relative vorticity when measured at 300 mb should amount to at least 5 x 10^{-5} sec^{-1} (10° lat)$^{-1}$ to serve as basis for rainfall computation.

Neglecting the Coriolis parameter, we have finally

$$w_h = -\frac{1}{\rho_h}\int_h^\infty \frac{\rho}{\zeta} V \frac{\partial}{\partial s}\left(KV - \frac{\partial V}{\partial n}\right) dz. \tag{7}$$

It is seen that in this expression nothing can change the sign of w_h except reversals of the vorticity gradient. As stated initially, we desire to find the areas in which precipitation will occur, not the precipitation amounts. Equation (7) would not be suitable for the latter purpose anyhow for various reasons. It is merely our hypothesis that some precipitation generally reaches the ground in areas where w_h is positive. We shall come back to this later on. At any rate, since we do not wish to find the magnitude of w_h, we can say that, provided the vertical integral can be represented by conditions on a suitable surface within the upper layer, in particular 300 mb,

$$\text{sign of } w_h = -\text{sign of } \frac{\partial}{\partial s}\left(KV - \frac{\partial V}{\partial n}\right)\Big]_{300\,mb}. \tag{8}$$

Herewith we have arrived at an expression that is accessible to simple calculation. Given an upper air prognostic chart, we can make a vorticity analysis and then delineate the precipitation areas. Cloudiness and precipitation should prevail in regions where the vorticity decreases downstream along the streamlines. Fair weather should prevail where it increases downstream along the streamlines. Here it should be noted that equation (8) involves only the vorticity gradient, not the sign of the vorticity itself. We can obtain the same gradient by a change from anticyclonic to cyclonic vorticity; by a change from weak to strong cyclonic vorticity; and by a change from strong to weak anticyclonic vorticity.

The next section outlines a practical routine for preparation of the vorticity analysis.

Vorticity computation and analysis: Considerable portions of observed upper air charts correspond to frequently repeating models (11) from which the forecaster can deduce the distribution of clear and cloudy areas at a glance. Other regions, however, deviate from the simple models perfected to date. In practice, it is easiest to compute the vorticity distribution for the whole map without regard to models. Evaluation of the field of geostrophic vorticity is not sufficient for this purpose in the high troposphere. The forecaster requires a streamline-isotach prognosis (5) as basis for his calculations. Given such a prognosis, the vorticity analysis can be made with the following six steps.[1]

(1) On each streamline, mark off the segments that have straight flow, cyclonically curved flow, and anticyclonically curved flow.

(2) Evaluate kV, best done with the aid of a transparent overlay (Fig. 1). This consists of concentric arcs with radii ranging from 250 km to 3000 km, drawn to fit the map scale and projection used.

- (a) Set the overlay over the base map and superimpose the arc which gives the best fit to the streamline curvature through the point in question;
- (b) read the value of the radius for the appropriate latitude;
- (c) read the value of V directly from the isotach analysis;
- (d) enter the upper portion of figure 2 with these quantities and read kV in units of $10^{-5} sec^{-1}$.
- (e) write the value obtained next to the point at which it applies, marking all cyclonic vorticities with a + sign and all anticyclonic vorticities with a - sign.
- (f) repeat the procedure at frequent intervals along each streamline.

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[1]Figure 1 has been constructed by C. O. Jenista, figure 2 by H. Riehl and C. O. Jenista. The method for preparing the vorticity computation largely follows instructions prepared by C. O. Jenista.

It is convenient to circle the values obtained.

(3) Now compute $-\frac{\partial V}{\partial n}$ at all points for which we have determined kV and also at frequent intervals in areas of straight streamlines.

(a) Place the appropriate Δn interval of 2.5° latitude normal to the streamlines and centered on the point to which the computation applies;

(b) from the isotach analysis read the velocity difference over the distance Δn;

(c) enter the lower left portion of figure 2 with the shear value and read $-\frac{\partial V}{\partial n}$ following the horizontal lines to the right hand margin, again in units of 10^{-5} sec^{-1}. More simply, we also can divide the velocity difference by 5; for instance, if the speed is 100 knots at one end of a 2.5° latitude interval and 50 knots at the other end, $|\Delta V|$ = 50 knots, and $\left|\frac{\partial V}{\partial n}\right|$ = 10×10^{-5} sec^{-1};

(d) enter the values of $-\frac{\partial V}{\partial n}$ on the map at the computed points, again appropriately marking them + for cyclonic and - for anticyclonic shear.

(4) Determine the relative vorticity at all points. Where we have values for kV or $-\frac{\partial V}{\partial n}$ only, these give the relative vorticity. At points where we have computed values of shear and curvature, we add these. The sign of the larger term determines the sign of the relative vorticity in cases where the two terms give opposing contributions.

(5) Draw isolines of equal relative vorticity.

(6) Shade the areas where the vorticity decreases along the streamlines. Here we should expect precipitation. Everywhere else we should expect no precipitation.

After some practice, a forecaster can carry out the complete analysis for the United States in 30 minutes.

Sample computation: Figure 3 illustrates a typical streamline-isotach pattern. We shall make sample computations at points A and B.

Example 1 at point A:

(a) With the overlay (Fig. 1) determine the radius of curvature -- 1200 km;

Note the velocity at the point -- 60 knots; enter the upper portion of figure 2 following the line 60 knots and read the value of kV = 2.4 units at the intersection with the interpolated horizontal line for r = 1200 km.
Determine the sign -- negative for anticyclonic curvature -- and enter value of -2.4 at A.

(b) Again using the overlay (Fig. 1) note the isotach difference over 2.5° latitude, taken perpendicular to the streamlines — 25 knots.
Enter the lower portion of figure 2 and along the interpolated horizontal line following ΔV = 25 knots read $\left|\frac{\partial V}{\partial n}\right|$ = 5 units.
This would also have been obtained by division of 25 by 5.
Determine the sign — negative for anticyclonic shear — and enter the value of -5 at A.

(c) Sum the two values and obtain -7.4 units, for practical purposes -7 whole units, for the value of the relative vorticity at A.

Example 2 at point B:

(a) Following the above routine, the radius of curvature is 720 km (cyclonic), the velocity 90 knots, and kV = +8 units.

(b) Assuming the speed at the jet axis to be 110 knots, the shear (cyclonic) is 35 to 40 knots through B. Using 40 knots, $\frac{\partial V}{\partial n}$ = 7.4 units.

(c) The relative vorticity at B is 15 units.

It is important to note that in the region of anticyclonic curvature north of the jet stream, the curvature terms are negative and the shear terms positive. In the region of cyclonic curvature south of the jet center, the curvature terms are positive and the shear terms negative. In those areas the terms give opposing contributions.

Near the level of maximum wind, values of cyclonic relative vorticity at times may exceed 30 units (3f at latitude 40°). Values of anticyclonic vorticity, however, should only rarely exceed 1f. Greater values would mean negative absolute rotation in space. The lower right hand diagram of figure 2 can be used for a quick check on the ratio of relative vorticity to Coriolis parameter at different latitudes.

Verification of equation (8) on observed maps: Since the method of precipitation forecast here proposed is based on a physical approach -- use of vorticity and continuity equations — it should become apparent quickly whether the simplifications made are serious. Sixteen actual cases have been computed for verification purposes, four in October, four in November and eight in December 1951. Considering that the evaluation covered the whole United States, this statistical sample which comprises 34 precipitation areas is quite large.

We proceeded as follows. At first, we shaded on the observed 300-mb charts (all 0400Z) the areas where precipitation should be occurring. We then superimposed with different shading the areas where precipitation (not including traces) did occur in the six hours ending at 0630Z.[1] We regarded

[1] As taken from the charts prepared at the Chicago Forecast Center of the United States Weather Bureau.

this six-hourly precipitation period which straddles the time at which soundings are taken as a fairer means of verification than an instantaneous picture. Where the areas of predicted and observed precipitation coincide, we have verification. Figure 4 reproduces all computed charts for the reader's own judgment. In our opinion, they are sufficiently successful to warrant recommending the method. Several comments follow:

(1) In some cases, verification is good in parts, but extensive precipitation also occurred that was not predicted. This precipitation (marked dotted in figure 4) consisted of showers in polar continental air surging southward to the rear of a cyclone, while the frontal precipitation was confined to the region of verification. We had previously suspected -- and it was herewith proven -- that our method may not be able to pick up such shower activity which is largely the result of thermodynamic instability in the lowest levels. The precipitating clouds may nowhere extend beyond the 850-mb level and the compensation for organized upward motion near the ground, if any, takes place well below 300 mb. Local effects, such as showers on the lee side of lakes, fall in the same classification. On the other hand, the effect of large-scale topographic features, such as the Rocky Mountains as a whole, should be reflected at 300 mb.

(2) Our treatment has assumed that precipitation reaches the ground in all areas where the vertical motion is upward at the surface of non-divergence. This cannot be entirely true. The air entering a zone of upward motion from the rear will require some time to reach the condensation level. If the moisture content is very low or the number of nuclei in the air insufficient or too large, precipitation may not reach the ground. Again, ahead of general rain or snow areas, virga and precipitation aloft may be observed over considerable distances. All this suggests that the computed precipitation areas should be too large, and to some extent this turned out to be true. Future work should take this into account. At this time, it is perhaps most important to emphasize that the omission of all moisture parameters apparently has proved to introduce only minor errors, contrary to fears voiced at the start of the study.

(3) According to the initial discussion, the precipitation on our charts should have occurred primarily in areas of pressure fall. We checked this point with the use of 12-hour pressure change charts. No one will be surprised to hear that the verification was very good.

(4) We add the following concerning some of the maps of figure 4.

October 21	Montana and southwest Canada. No vorticity analysis due to lack of data.
October 22	Data in Pacific Northwest sufficient for analysis in this case and verification obtained.
November 16	Poorest map by far. A large area of very light snow lies under a 300-mb trough extending southwestward from the Great Lakes. Vorticity gradients are very weak. This case requires further study.
November 25	U. S.-Canadian border in northwest: Scarce data and analysis doubtful.

November 27	Large precipitation area in northeast indicated to limit of forecast area.
December 5	Oregon and Washington: Failure probably due to inability to locate jet center off West Coast.
December 7	Vorticity gradient weak over Rockies. Precipitation scattered with clear skies reported by neighboring stations.
December 14	Very few observations near Montana-Canadian border. Vorticity analysis uncertain.
Principal Failures	General - November 16 East of Salt Lake - November 25 New York - December 5

Prognostic Precipitation Patterns: Prognostic 24-hour precipitation charts were prepared for 14 days during November and December 1951, based on upper wind prognoses prepared by Dr. F. Defant, C. O. Jenista and R. Renard. Figure 5 shows all prognostic charts made. The verification secured was remarkably good, especially if it is considered that all the work was in the experimental stage, including the wind forecast. The reader will note that the verification progressively improved from beginning to end of the trial period. Some comments follow.

November 28	Southeast: Latitude of jet center predicted too far north. Central U. S.-Canadian border: "Over-forecast"; lack of experience on first prognosis.
November 29	Northeast: On fringe of forecast area. Probably should not have been entered.
November 30	"High index" map with weak systems. Indiana and Ohio: Trough passed with much cloudiness but no measured rain. Vorticity field weak. Colorado: Considerable cloudiness, weak vorticity field.
December 1	High index trough with much cloudiness west of Great Lakes.
December 4	Northwest Pacific and California outside forecast area. General poor verification due to failure of wind prognosis.
December 6	Southeast: Error in wind forecast.
December 7	Limit of prognosis at 105°W.
December 8	Vorticity gradients weak in north near 100°W.
December 11	New Mexico: Discrepancy due to rapid inward displacement of jet center from Pacific Ocean.

Southeast: Failure due to rapid advance of jet from Mexico, outside data.

December 12 — Northwest: No data for reliable prognosis. Southern California: Predicted correctly by alternate forecaster who brought jet center in at coast from Pacific.

December 14 — No prognosis made for northwest — lack of data. Arizona: Vorticity gradient weak. Apparently some verification.
West-central U. S: Computed precipitation furnished important clue for rapid development of major snowstorm, predicted to form later with other considerations.

December 15 — Large precipitation area west of Great Lakes pointed out as unlikely at time of forecast. Difficulty in locating jet center coming in from Canada due to sparse data.
Eastern U.S.: Split precipitation area did not verify, but formation of secondary storm in southeast was suggested, and this occurred.

December 18 — No forecast made for northwest.

Conclusion: The results of this study, covering the verification of 16 observed and 14 prognostic charts, are sufficiently impressive in our opinion to warrant continued work with this method. We hope that it will stimulate attempts to go from qualitative to quantitative precipitation forecasts at forecast centers. We also wish to point out that the method is not restricted to 24-hour prognoses, but will apply to any time interval for which upper wind predictions are made.

In order for the forecasts to verify, good prognostic 300-mb charts must be available. Although the frequency of wind observations at this level has increased considerably in recent years, there is still a dearth of winds on many days. If our results are accepted, it follows that high-level observations are of far greater importance for predictions for the general public than believed to date. For this reason we would like to recommend that the network of rawins over the United States should be increased to at least the density of the radiosonde network; that the agencies responsible for observations make every effort possible to obtain all 300-mb or 30,000-foot winds at all times; and that the winds and radiosonde data for the high levels be transmitted at the earliest possible moment, therewith eliminating the stigma of "second transmission." Finally, the reports of high-altitude aircraft, now completely inaccessible, should be placed at the disposal of the forecasters.

Acknowledgment

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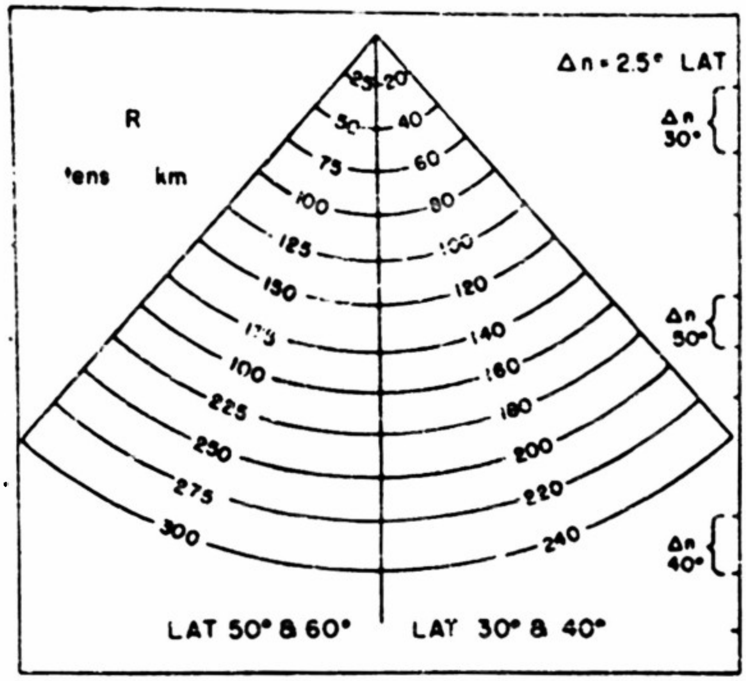


Figure 1. Overlay to determine radius of stream-line curvature in tens km. Δn intervals on right margin of overlay give distance corresponding to 2.5° latitude for different latitudes. All numbers are applicable to base map in use at University of Chicago. A corresponding overlay can be drawn up for any map scale and projection.

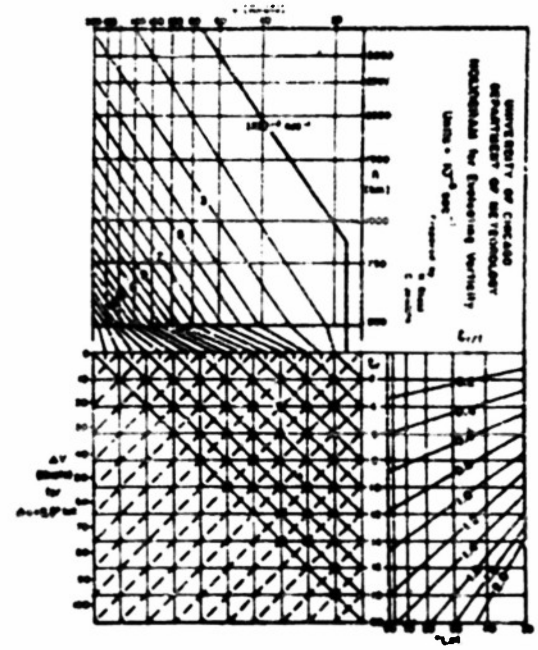


Figure 2. Nomogram for computation of relative vorticity. Text contains directions for use.

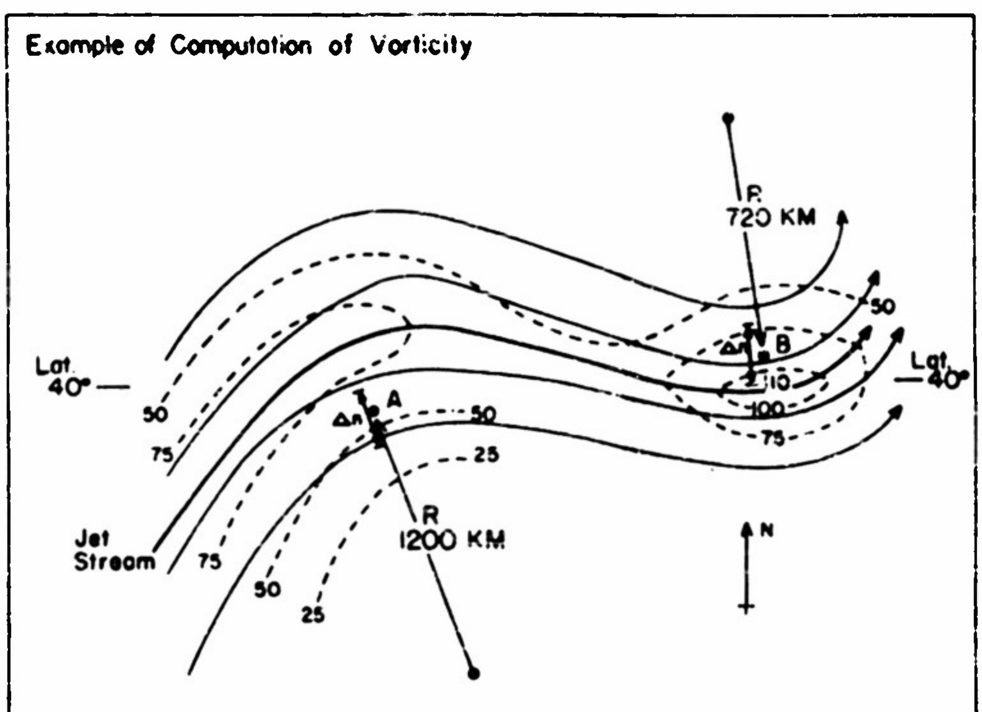
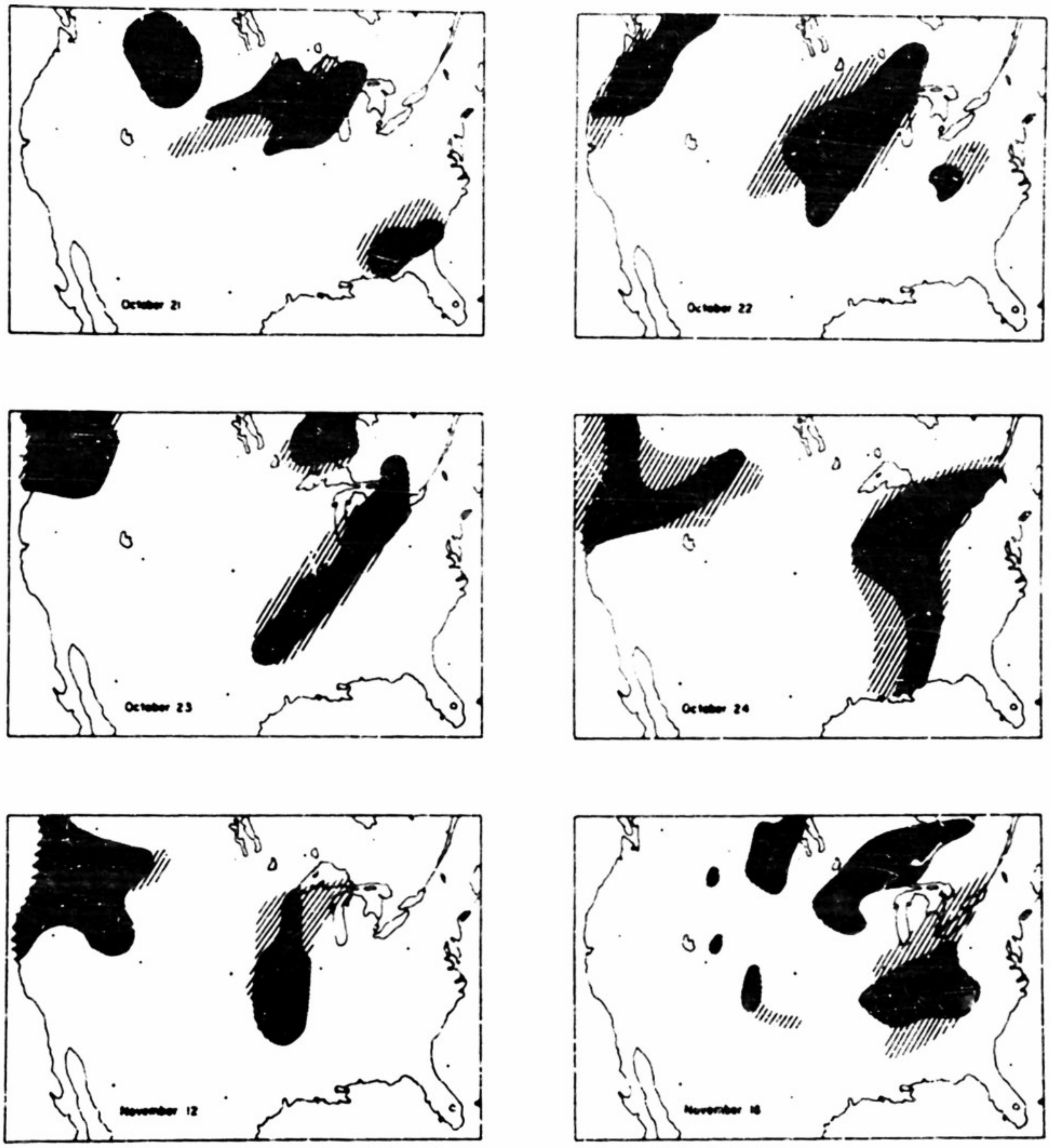


Figure 3. Map for sample computation of relative vorticity carried out in text.

Figure 4

Areas where relative vorticity decreased downstream at 300 mb on sixteen observed maps October to December, 1951, 0400Z (shading oriented NE-SW); and observed six hourly precipitation ending at 0630 for the same days (shading oriented NW-SE for frontal precipitation, dotted for low level showers during polar outbreaks and lake precipitation). Verification is obtained where shadings overlap.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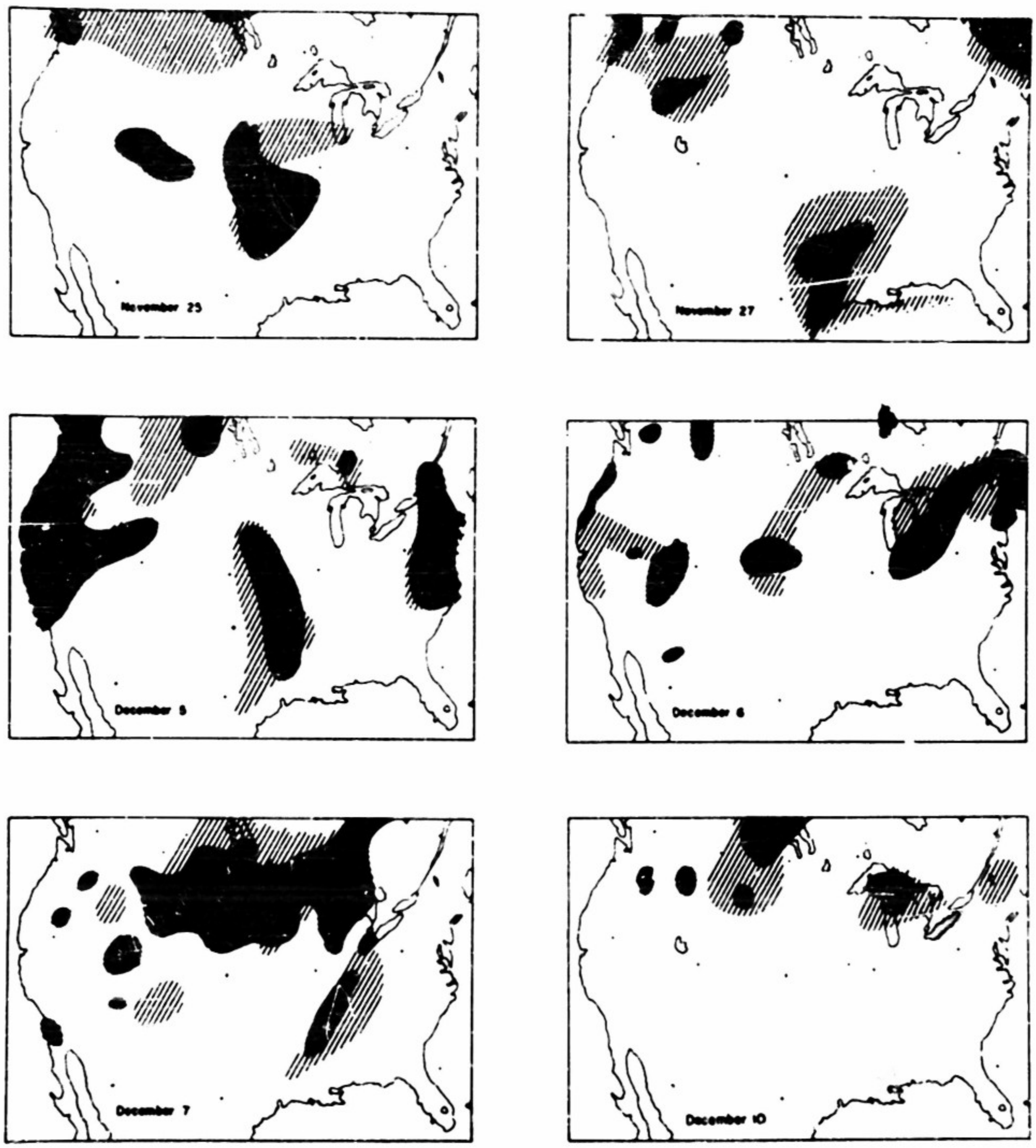
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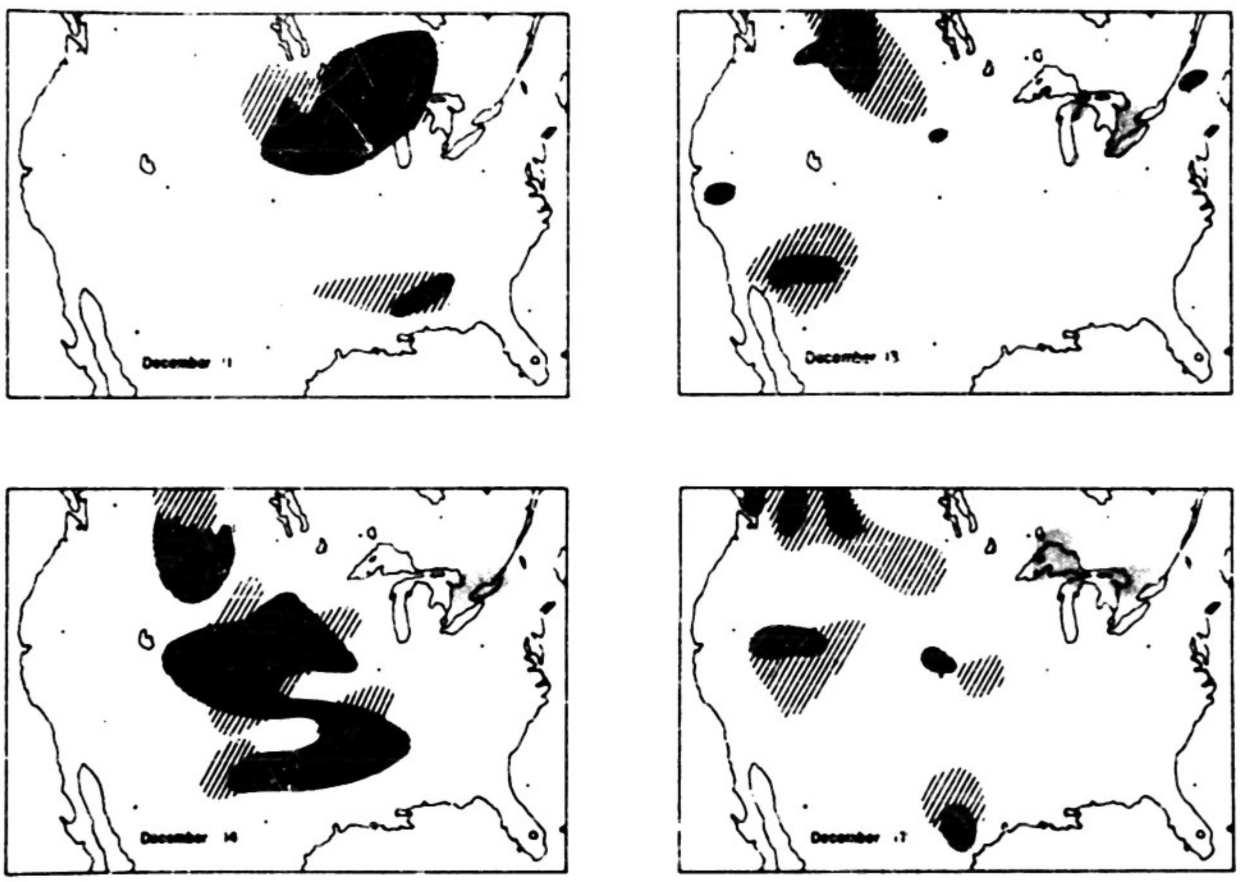
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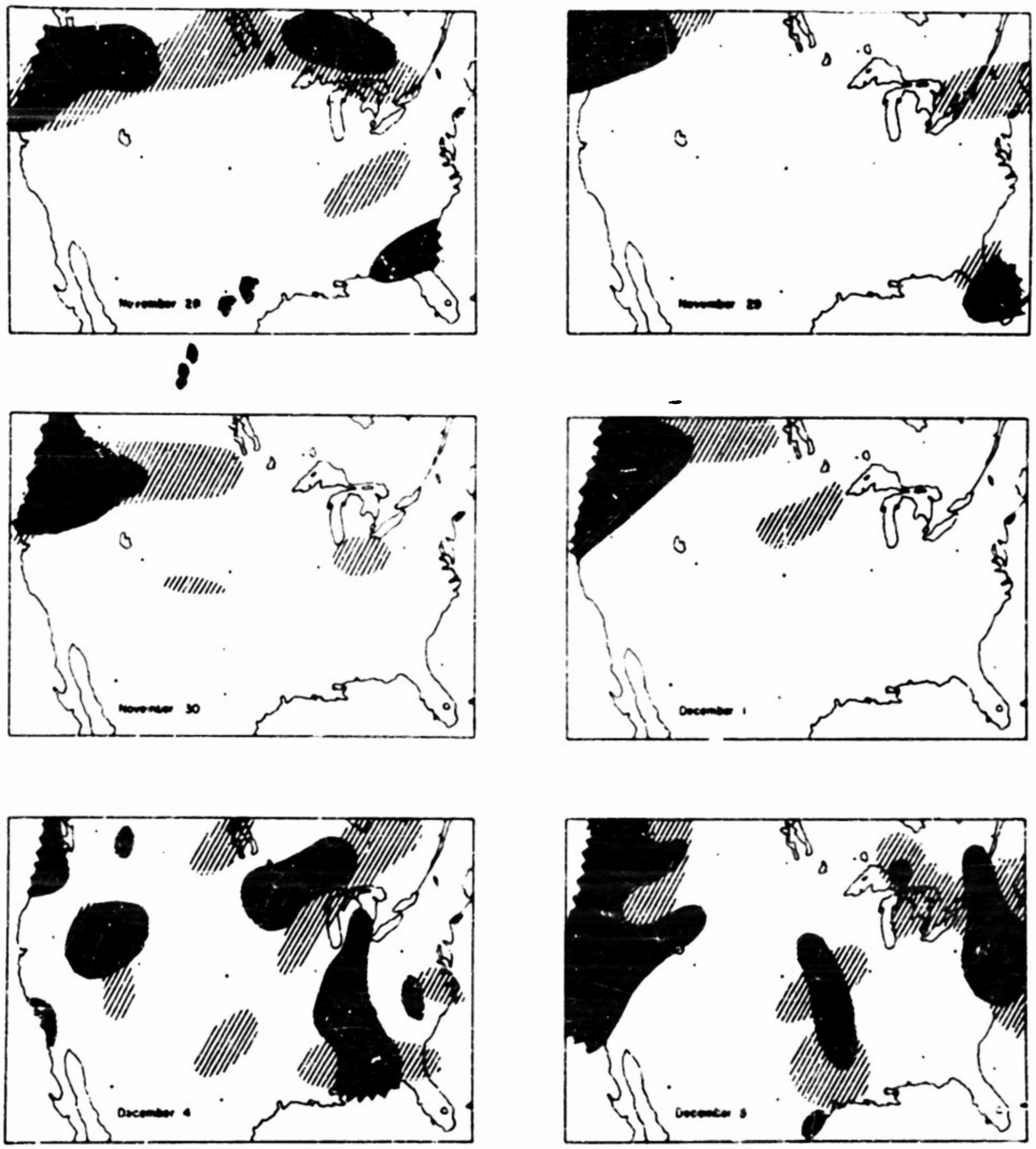

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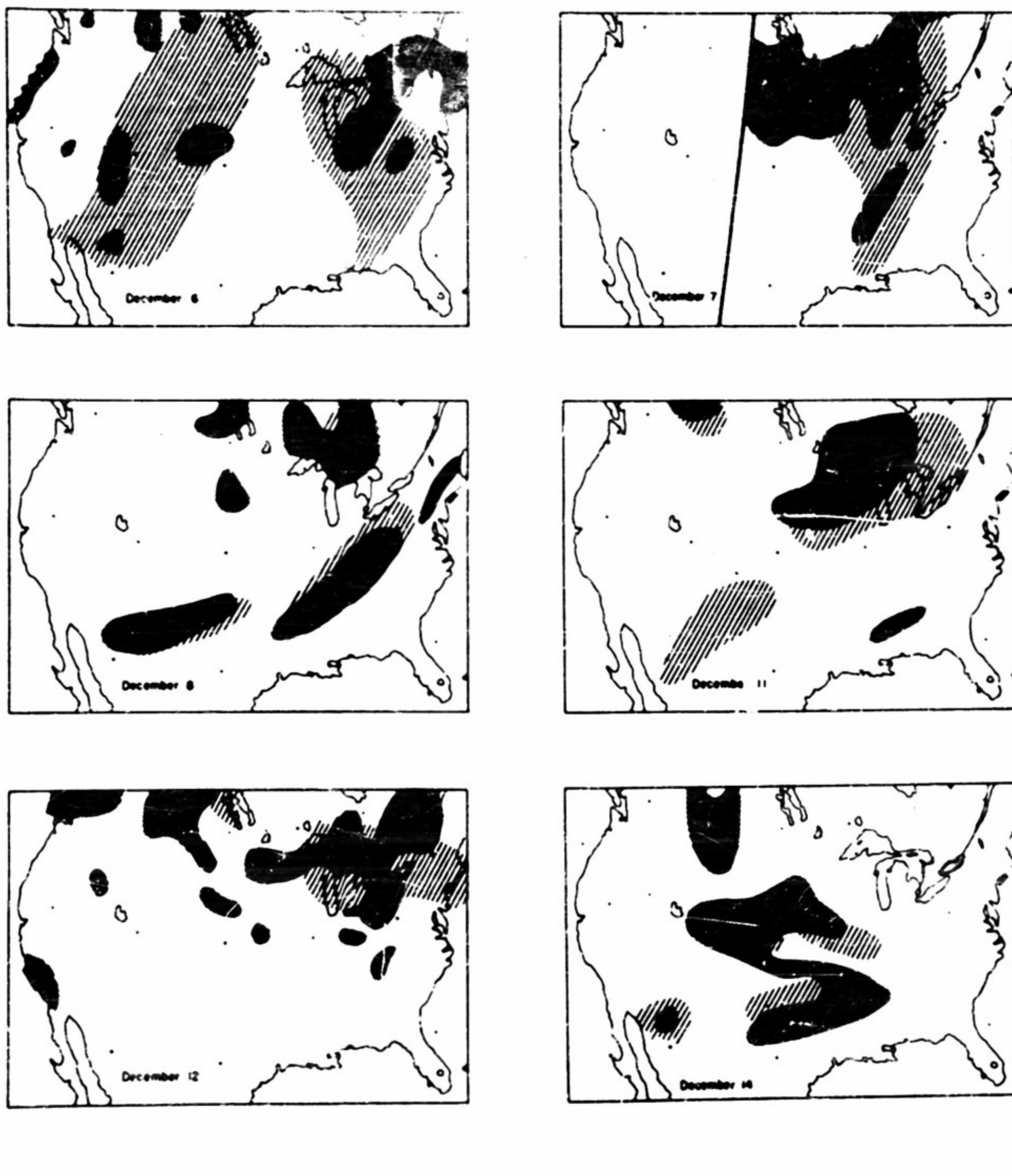
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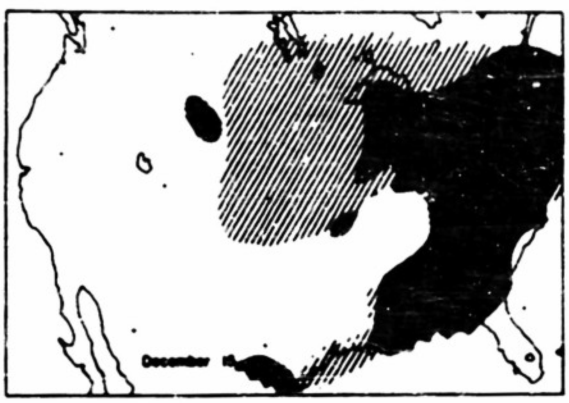
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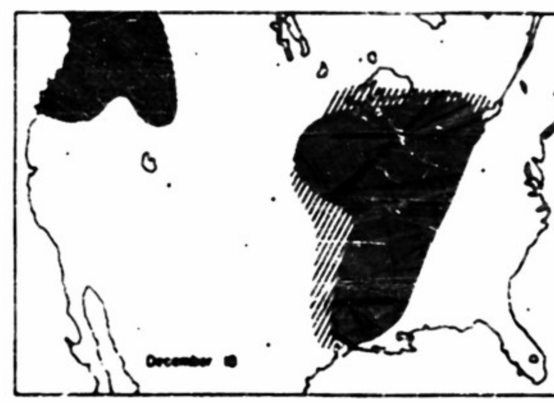
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